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ABSTRACT

This study explores seventh- and eighth-grade students' thinking about mathematical patterns. Interviews were conducted in which students solved problems about sequential perimeter and area problems modeled with pattern blocks and tiles, generalized the relationships related to the patterns and represented the relationships symbolically, identified other valid symbolic expressions of the pattern, and encountered equation-evoking situations. Research questions pertained to the strategies middle school students use to reason when solving pattern problems, symbolic representations the students develop, the students' interpretations of symbolic representations, and the students' strategies for solving equation-evoking situations. The results of this study support the use of mathematical patterns to promote algebraic reasoning and provide descriptions of middle school students' reasoning as they engage in solving a specific type of pattern problem. Findings also suggest that experience exploring the relationships in sequential perimeter and area patterns may help students develop an appreciation for the meaning of expression. Contains 16 references. (DDR)

Middle School Students' Understanding of Mathematical Patterns and Their Symbolic Representations

by

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Purpose of the Study

The far-reaching changes which have swept through American society during the past century have created a need for a redefinition of mathematical competence (National Council of Teachers of Mathematics, 1989). The changing workplace now requires workers with "mathematical power" based on the abilities to explore, conjecture, and reason logically, and the ability to use a variety of mathematical models effectively to solve problems (Committee on the Mathematical Education of Teachers of Mathematics, 1991). Calculational skills are not sufficient; instead students must reason effectively about quantities and quantitative relationships (Thompson & Thompson, 1995). Algebra provides the concepts and language that facilitate reasoning about relationships within problematic situations. The content of algebra has such broad applications that it functions as a gatekeeper which regulates access to the symbolic and abstract mathematics essential to an increasingly wide range of careers (Christmas & Fey, 1990; Phillips, 1995). Economic and equity issues compel mathematics educators to face the challenge of making the ideas of algebra accessible to a broader segment of the population (Lawson, 1990).

Kaput (1995) has recommended that algebra should be reconceptualized as a strand woven through many grade levels, serving as a sense-making tool in elementary and middle school. In the Curriculum and Evaluation Standards for School Mathematics (CESSM) (1989), the National Council of Teachers of Mathematics (NCTM) recommends that experiences with patterns and relationships provide an introduction to algebraic concepts in grades K-4 and be extended to focus on analysis, representations and generalizations of function relationships in grades 5-8. As students study relationships among the quantities in a pattern, they construct knowledge about important mathematical concepts such as functions, and learn to reason and communicate about the important content and processes of algebra (Phillips, 1995).

If the study of patterns is a valid means of preparing students for algebra, then more information is needed describing how children think about patterns. The purpose of this study was to explore seventh and eighth-grade students' thinking about patterns by conducting task-based interviews in which the students were (a) asked to solve problems about sequential perimeter and area problems modeled with pattern blocks and tiles, (b) generalize the relationships related to the patterns and represent the relationships symbolically, and (c) identify other valid symbolic expressions of the pattern. The students also (d) encountered equation-evoking situations, situations which a student with knowledge of formal algebra might address by solving an equation. The research questions were as follows: What are the strategies that middle school students use to solve pattern problems involving perimeter and area? What are the relationships among the following: (1) the strategies middle schools students use to reason about pattern problems, (2) the symbolic representations the students develop, (3) the students' interpretations of symbolic representations, and (4) the students' strategies for solving equation-evoking situations?

Theoretical Frameworks

The concept of quantitative reasoning as described by Thompson (1993, 1994) and Thompson and Thompson (1995) provided the theoretical framework for the design of this study. True algebraic power requires an awareness of underlying relationships and should be built on a foundation of quantitative reasoning (Thompson & Thompson, 1995). This reasoning is developed in situations where students reason about quantities and quantitative relationships, and use arithmetic notations to represent their reasoning (Thompson, 1993). The design of the study was also guided by Carey's (1991) study which found that the domain of numbers in problems influenced the problem-solving strategies used by children and that the use of alternative number sentences provided a revealing context for studying children's thinking.

Two models provided structure for describing children's thinking about the pattern problems in the study. Thompson, Philipp, Thompson and Boyd (1994) described two sharply contrasting orientations toward mathematics: conceptual and calculational viewpoints. Teachers and students who possess a conceptual orientation focus on the aspects and relationships of situations that give meaning to numerical values and that suggest numerical operations. Teachers and students who approach mathematics with a calculational orientation emphasize procedures as a means of getting answers. From a conceptual viewpoint, reasoning is of utmost importance; from a calculational perspective, reasoning may be irrelevant compared to the value of the numerical answer. The conceptual orientation more readily supports quantitative reasoning.

The other model resulted from research into children's solution strategies on simple addition and subtraction problems (Carpenter & Moser, 1984). Children's strategies on pattern problems in this study correspond somewhat to those described in the previous research on addition and subtraction. Carpenter and Moser demonstrated that children use combinations of strategies that become increasingly abstract as their understanding develops. The middle school students in this study also appeared to shift from less abstract to more abstract strategies as their understanding of the pattern relationships developed.

Methods of Inquiry

This study followed the naturalistic paradigm of qualitative research (Merriam, 1988; Miles & Huberman, 1994; Stake, 1995) in the respect that the intent of the research was to develop an integrated understanding of students' reasoning about pattern problems (Miles & Huberman, 1994). The emergent research design (Stake, 1995) was appropriate for this situation because there exists no model for describing children's thinking about patterns. Furthermore, the emergent design allowed for adjustments to accommodate information which developed in the course of the study. This study was conducted in three phases. In Phase 1, a tentative group of problems was compiled and tested in four interviews. Information from the Phase 1 interviews guided revision of the interview protocol, which was tested with 17 seventh- and eighth-grade students and further refined in Phase 2. The protocol was expanded for Phase 3 to include three sequential pattern problems and one sequential area problem. Twenty-three seventh- and eighth-grade students participated in Phase 3.

For each problem the students were shown the first four figures of the sequence and asked to predict the perimeter or area for several figures from later in the sequence. Next students were asked how they would tell someone how to find the perimeter or area of the figure no matter what the number of the figure was, in effect a request for a generalization of the pattern. Students were then shown a series of cards and asked whether or not the expressions on them described the patterns in the problems. Students also encountered an equation-evoking situation for each problem in which they were given a perimeter or an area and asked to find the number of the figure. At every step students were asked to explain their reasoning. All written work completed by the students was collected and saved.

Data Sources

The interviews were audio- and video-taped and later transcribed. The transcribed notes and the students' written work were coded according to strategies used by the students, accuracy of outcomes, and implications for student understanding. The strategies used by the middle school students on each of the four components of the interview were analyzed separately, and then the relationships among strategies across the interview components were also analyzed.

Results

The students in this study used five distinct strategies to find perimeter or area of figures that were not modeled for them: (a) Model the figure and count the block-sides;

(b) Multiply the number of the figure by two, three, or four, an in appropriate strategy because two operations were needed to find perimeter or area; (c) Apply Proportional Reasoning, another inappropriate strategy; (d) Skip Count/Add, a strategy based on the perimeter or areas of consecutive figures; and (e) Use an Expression or an Equation, a strategy based on the relationships between the number of the figure and its perimeter or area.

The three main strategies for identifying alternative symbolic expressions were: (a) Substitute a number into the expression and compare the results to a known perimeter or area; (b) Compare to Another Expression, in other words, consider the equivalence of the expressions; and (c) Relate the Expression to the Model, or base a judgment on the relationships in the figures. Some students also (d) Applied Vague, Non-Quantitative Reasoning about whether an expression would yield quantities that were too large or too small, and some students (e) Interpreted the expressions Inappropriately in their struggle to attach meaning to the expressions.

On the equation-evoking situations, the students used eight different strategies. The students who used a (a) Model or (b) Guess and Check strategy avoided reasoning about the quantities in the relationships. Some students misinterpreted the relationships in the figures when they tried unsuccessfully to (c) Divide or Multiply, or (d) Apply Proportional Reasoning. The students who (e) Skip Counted recognized the relationships between the perimeters of consecutive figures, and the students who (f) Observed Relationships in the Figures, (g) Worked Backwards, or (h) Solved Equations had identified important relationships between the number of the figures and its perimeter or area.

Three separate clusters of students were identified. A number of commonalities within and between clusters existed, but the single factor which consistently distinguished the members of each cluster from the others was the type of symbolic representations the students developed to describe the patterns in the problems. Identified according to the type of symbolic representations they developed, the three clusters were Verbal and Single-Operation Expressions and Equations, Transition from Verbal and Single-Operation to Symbolic Expressions, and Equations and Symbolic Expressions and Equations.

When asked to explain how to find perimeter or area, the seven students in the Verbal and Single-Operation cluster typically said to "count all the sides" or wrote an expression incorporating one operation such as 4 x n although the all patterns in the problems required two operations to adequately describe them. None of the students in this cluster wrote a valid expression to represent the relationships in the pattern situations. This cluster included all the students who modeled figures to find perimeter or area and none of the students who used equations to solve equation-evoking situations. Their success rate on equation-evoking situations was low, and their few correct answers relied on Model strategies or on Skip-Count and Add strategies which indicated an understanding of the relationship between the perimeters of consecutive figures instead of the relationships between the number of a figure and its perimeter or area. The Verbal and Single-Operation group often operated from a calculational perspective although some students provided evidence that they were searching for the meaning of expressions.

The six students in the Transition from Verbal and Single-Operation to Symbolic Expression or Equation cluster began with a verbal or single-operation expression but eventually wrote a concise and appropriate symbolic expression for at least one problem. To find perimeter and area, all but one student in the group used an expression or an equation based on a number pattern at least once. The students in this group correctly identified nearly all alternative expressions using Substitute and Compare to Another Expression strategies. Their strategies on equation-evoking situations ranged from immature and inaccurate to accurate and precise. They were more like the Verbal and Single-Operation cluster on their first problems and more like the Valid Symbolic Expression or Equation cluster on their last problems.

The ten students in the third cluster wrote concise Valid Symbolic Expressions or Equations to represent all the patterns in this study. They were mostly successful at finding perimeter and area. The students in this cluster used a combination of Substitute, Compare to Another Expression, and Relate the Expression to the Model strategies to identify alternative expressions with a high degree of success. They solved all but one of the equation-evoking situations correctly, either with a formal algebraic solution or by working backwards. The students in this group were most likely to operate from a conceptual orientation, developing a expression or equation or deciding about an alternative expression by considering the relationships in the figure.

The characteristics of the students in the three clusters suggest four levels of understanding of sequential perimeter and area problems. At the most concrete level, students model the figures and count block-sides to find perimeter or area and to solve equation-evoking situations. When asked to generalize a method for finding perimeter or area, students at this level give verbal directions for counting the sides of the figure. They struggle to interpret symbolic expressions and succeed mostly by substituting a number into an expression and comparing the results to a known perimeter. They demonstrate little or no awareness of the relationships between the number of the figure and its perimeter or area.

At the second level, students demonstrate an awareness that there exists a relationship between the number of the figure and its perimeter or area, but they misinterpret the relationship, expressing it in terms of a single operation. These students frequently attempt to find perimeter or area and solve equation-evoking situations by applying a single operation and reversing it with little success. Their awareness that there must exist a relationship presents a teachable moment.

Students at the third level interpret the patterns in terms of the relationships between the perimeter or area of consecutive figures. Students in this study who described an Add 2 or Add 3 pattern often found perimeter and area and solved the equation-evoking situations by skip counting by two or three, but they did not solve equations. However, several students who used the Add and Skip-Count strategies to find perimeter and area later recognized the relationships between the number of the figure and the perimeter or area, and began to operate at the fourth level of understanding.

Students at the fourth level of understanding recognize the relationships between the number of the figure and its perimeter or area, and express the relationships symbolically. In this study, students at this level were more likely to explain their calculations in terms of the relationships of the figures, and solved equation-evoking situations either informally by working backwards or with formal algebraic solution methods.

Educational Importance of the Study

The results of this study are useful to mathematics teachers and mathematics teacher educators who want to use mathematical patterns to promote algebraic reasoning. The findings of this study also provide justification for curriculum developers who create materials based on mathematical patterns to prepare students for algebra. The results provide descriptions of middle grade students' reasoning as they are engaged in solving a specific type of pattern problem, and the manner in which the students can relate the patterns to symbolic representation. The results suggest that experience exploring the relationships in sequential perimeter and area patterns may help students develop an appreciation for the meaning of expressions.

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